

KNOW	/ 12	APP	/ 12	INQ	/ 12	COMM	/ 6
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**MCV4UP - UNIT 2 – LIMITS AND CONTINUITY****TEST****GIVE ALL ANSWERS IN EXACT FORM, UNLESS STATED OTHERWISE.****PART A – This section is to be completed without the use of a calculator. Upon completing this section, hand it in to receive the remainder of the test.**1) Evaluate each of the following limits. All answers should be exact. (*K – 2,2,2,2,2 marks*)

a)  $\lim_{x \rightarrow 1} \frac{x^3 - 2x^2 + 4x - 7}{x - 2}$

b)  $\lim_{x \rightarrow -5} \frac{2x^2 + 7x - 15}{x^2 + 3x - 10}$

c)  $\lim_{x \rightarrow 4} \begin{cases} x^2 - 7, & x < 4 \\ \frac{1}{2}x + 7, & x \geq 4 \end{cases}$

d)  $\lim_{x \rightarrow 5} \frac{2x|x-5|}{x-5}$

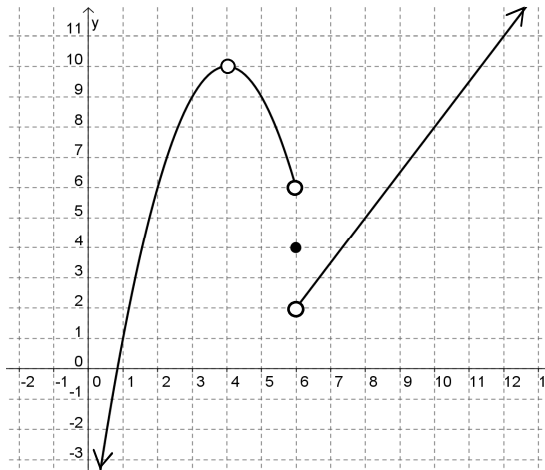
e)  $\lim_{x \rightarrow 11} \frac{\sqrt{x+5} - 4}{3x - 33}$

f)  $\lim_{x \rightarrow 25} \frac{x - 25}{\sqrt[3]{x+2} - 3}$

**PART B – This section may be completed with the use of a calculator.**

NAME: \_\_\_\_\_

- 2) The graph of  $y = f(x)$  is shown below. Determine the value of each of the following or indicate that it does not exist. (A – 4 marks)



a)  $\lim_{x \rightarrow 6^-} f(x) = \underline{\hspace{2cm}}$

b)  $\lim_{x \rightarrow 4} f(x) = \underline{\hspace{2cm}}$

c)  $f(6) = \underline{\hspace{2cm}}$

d)  $\lim_{h \rightarrow 0} \frac{f(9+h) - f(9)}{h} = \underline{\hspace{2cm}}$

- 3) Use the difference quotient method to determine the **equation** of the tangent to the graph of  $f(x) = \sqrt{x+5} - 7$  where  $x = 4$ . Express your answer in exact form. (A – 4 marks)

- 4) A clinical test was conducted of a new drug designed to eliminate microbes. The results of a patient's blood samples showed that the number of microbes per millilitre,  $M$ , at  $t$  hours after taking the drug is  $M(t) = 40000(36 - t)^2$ ,  $t \geq 10$ . Determine the rate of change in the number of microbes per millilitre 15 hours after the drug is taken. (A – 4 marks)

5) Evaluate the following limits. (*I – 2 marks*)

a)  $\lim_{x \rightarrow -\infty} \frac{8x - 3x + 1}{x^2 + x - 1} = \underline{\hspace{2cm}}$

b)  $\lim_{x \rightarrow \infty} \frac{10x^3 + \sin(2x)}{5x^3 - \cos(2x)} = \underline{\hspace{2cm}}$

6) Evaluate  $\lim_{x \rightarrow 0} \frac{\sin(4x) + 2\sin(3x)}{x}$ . (*I – 2 marks*)

7) Use the formal definition of a limit to prove that  $\lim_{x \rightarrow 5} (3x + 2) = 17$ . (*I – 5 marks*)

8) An oil tank is being drained. The volume  $V$ , in litres, of oil remaining in the tank after  $t$  minutes is given by  $V(t) = 60t^2 - 3000t + 37500$ , where  $0 \leq t \leq 25$ . After how many minutes is the oil draining at exactly  $-1200$  L/minute? (*The rate is negative since the volume is decreasing.*) (*I – 3 marks*)

9) Use the Sandwich Theorem to show that  $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$ . ( $C - 2$  marks)

10) Karlo and Jenny were enjoying a conversation about the foundations of calculus. During their discussion, Jenny told Karlo that the expression  $\frac{f(2+h) - f(2)}{h}$  is the slope of the tangent line to  $f(x)$  at the point where  $x = 2$ . Is Jenny's claim correct? Explain. ( $C - 1$  mark)

11) The function shown below possesses three types of discontinuity. Specifically, it exhibits a vertical asymptote, a hole and a jump discontinuity. State the values at which each of these discontinuities occur. ( $C - 3$  marks)

$$f(x) = \begin{cases} \frac{x^2 + 5x + 6}{x^2 - 2x - 8}, & x < 1 \\ \sqrt{x} + 8, & 1 \leq x \leq 4 \\ \frac{x^2 - 16x + 28}{x^2 - 7x + 10}, & x > 4 \end{cases}$$

Vertical asymptote at  $x =$  \_\_\_\_\_

Hole at  $x =$  \_\_\_\_\_

Jump discontinuity at  $x =$  \_\_\_\_\_