UNIT 7 – APPLICATIONS OF VECTORS
SUPPLEMENTARY REVIEW

1) Two forces of magnitude 27 N and 42 N act at an angle of 115° to each other. Determine the magnitude of the **equilibrant** and its direction with respect to the 27 N force.

2) A plane flies on a heading of 040° at a speed of 500 km/h. A wind blows **from** the south-east at 55 km/h. Calculate the magnitude and direction of the plane's groundspeed.

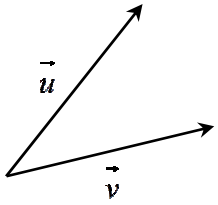
3) If $\vec{u} = (-1, 7)$ and $\vec{v} = (0, 6)$, find $-3\vec{u} \bullet (2\vec{v} - \vec{u})$.

4) Find the angle between the vectors (7, 5) and (4, -2).

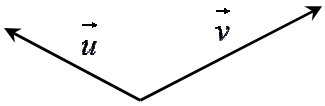
5) Show that there is no value k such that the angle between the vectors (3, 4) and (0, k) is 60°.

6) For each of the following diagrams, draw $\vec{u} \downarrow \vec{v}$.

a)



b)



7) If $\vec{a} = (12, 4)$ and $\vec{b} = (1, -1)$, determine the projection of $\vec{a}$ on $\vec{b}$.

8) Vectors $\vec{u}$ and $\vec{v}$ are such that $|\vec{u}| = 9$, $|\vec{v}| = 5$ and the angle between them is 45°. Determine the following:

a) $\vec{u} \bullet \vec{v}$ b) $|\vec{v} \downarrow \vec{u}|$

9) A box is pushed up a 10 m ramp with a force of 25 N applied at an angle of 45° to the horizontal ground. The box is then pushed along a horizontal platform for 12 m using the same force. If the ramp makes an angle of 15° with the ground, find the work done.

10) Prove that $\vec{u} \bullet \vec{u} = |\vec{u}|^2$ for any vector $\vec{u}$.

11) Explain why the dot product of two perpendicular vectors is 0.

12) Explain why the Associative Law, $(\vec{u} \bullet \vec{v}) \bullet \vec{w} = \vec{u} \bullet (\vec{v} \bullet \vec{w})$, is meaningless for the dot product.

13) State whether the following expressions are vectors, scalars or meaningless (operations in brackets are completed first).

a) $(\vec{a} \bullet \vec{b}) + (\vec{b} \times \vec{c})$
b) $\vec{a} \bullet (\vec{b} \times \vec{c})$
c) $(\vec{a} \times \vec{b}) \bullet (\vec{b} \times \vec{c})$
d) $(\vec{a} \bullet \vec{b}) \times \vec{c}$
e) $(\vec{a} \bullet \vec{b})\vec{c}$

14) For the vectors $\vec{a} = (5, 1, -2)$ and $\vec{b} = (3, -2, 7)$, determine each of the following.

a) $\vec{a} \bullet \vec{b}$
b) $\vec{a} \times \vec{b}$
c) $\vec{a} \downarrow \vec{b}$
d) $\left|\vec{a} \downarrow \vec{b}\right|$

15) Find m if $(m, -2, 5)$ and $(3, 2m, -5)$ are perpendicular vectors. *(3 marks)*

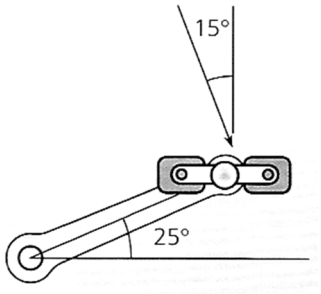
16) Two vectors $2\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ are perpendicular. Find the angle between $\vec{a}$ and $\vec{b}$, if $\left|\vec{a}\right| = 2\left|\vec{b}\right|$.

17) Determine if the vectors $\vec{a} = (2, -1, 4)$, $\vec{b} = (3, 1, 4)$ and $\vec{c} = (1, -3, 4)$ are coplanar.

18) Determine the direction angles of the vector $\vec{u} = 3\vec{i} + 4\vec{j} - 5\vec{k}$.

19) The points $A(5, 3, 6)$, $B(2, 7, 9)$ and $C(8, 1, 7)$ are three vertices of a triangle. Find the area of this triangle.

20) Find the torque produced by a cyclist exerting a force of 115 N on a pedal in the position shown in the following diagram. The shaft of the pedal is 0.16 m long.



22) Given $\vec{u} = (a, b, c)$, $\vec{v} = (d, e, f)$ and $\vec{w} = (g, h, i)$, prove that $\vec{u} \times \vec{v} \bullet \vec{w} = \vec{u} \bullet \vec{v} \times \vec{w}$.

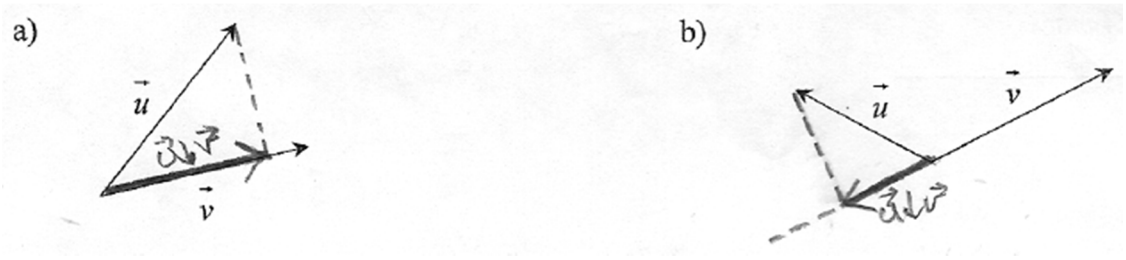
23) What is the minimum amount of work done in sliding a refrigerator 1.5 m across a kitchen floor against a frictional force of 150 N?

24) How much work is done by gravity in causing a 294 N rock to tumble 40 m down a hill at an angle of 38° to the ground?

25) If a 10 N force, acting in the direction of the vector (1, 1), moves an object from the point (-2, 1) to the point (5, 6), calculate the work done. The distance is in metres.

Answers

1) 39.2 N at an angle of 103.6° to the 27 N force.
2) 507.8 km/h on a heading of 033.8°
3) -102
4) 62.1°
6) a) b)



7) (4, -4)
8) a) $\frac{45\sqrt{2}}{2}$ b) $\frac{5\sqrt{2}}{2}$
9) 428.6 J
11) The angle between perpendicular vectors is 90°. Therefore, for perpendicular vectors $\vec{u}$ and $\vec{v}$, we have $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos 90°$, which equals 0, since cos90°=0.
12) The dot product of two vectors gives a scalar, which therefore cannot be dotted with a third vector.
13) a) meaningless b) scalar c) scalar d) meaningless e) vector
14) a) -1 b) (3, -41, -13) c) $\left(-\frac{3}{62}, \frac{1}{31}, -\frac{7}{62}\right)$ d) $\frac{\sqrt{62}}{62}$ $(\doteq 0.13)$
15) m = -25
16) 60°
17) coplanar
18) $\alpha = 64.9°$, $\beta = 55.6°$, $\gamma = 135°$
19) $\sqrt{70}$ units^2
20) 18.1 J
23) 225 J
24) 7240 J
25) $60\sqrt{2}$ J