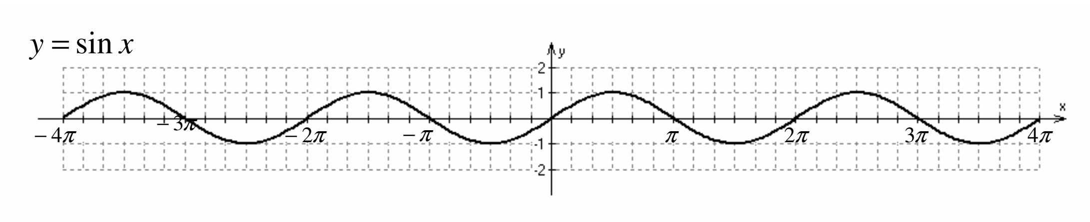
<u>SOLVING LINEAR TRIGONOMETRIC EQUATIONS</u>

Consider the following **trigonometric equation**:

$$\sin x = 1$$

If we were to solve this equation, we would be looking for all the values of x such that $\sin x$ works out to 1. How many solutions do you think exist? (Hint: Think of the sine graph!)



It doesn't take long to realize that there are infinitely many solutions! By looking at the graph, we see that $\sin x = 1$ when $x = \frac{\pi}{2} + 2n\pi$ (n is an integer). The possibility of an infinite number of solutions is a result of the periodic nature of the sine function. In the following equations, however, will restrict our attention to the domain $0 \le x \le 2\pi$.

Example

Solve $\cos x = -\frac{1}{\sqrt{2}}$, $0 \le x \le 2\pi$.

Solution

Begin by finding the related acute angle. In this case, the related acute angle is $\frac{\pi}{4}$ (or $45°$). We could find the related acute angle by using a special triangle or by using the $\cos^{-1}$ function on your calculator. In either case, notice that we **ignore the negative sign** in front of $\frac{1}{\sqrt{2}}$ when finding the related acute angle. The negative sign will be used to determine the quadrants in which the terminal arm can lie.

Since the sign of this cosine ratio is negative, the terminal arm lies in either quadrant II or quadrant III (CAST rule). A sketch is helpful:

Quadrant II Quadrant III

Therefore, the values of x, such that $0 \le x \le 2\pi$, are: $x = \frac{3\pi}{4}$ $(135°)$ and $x = \frac{5\pi}{4}$ $(225°)$.

Examples

Solve each of the following equations for $0° \le \theta \le 360°$ (or $0° \le x \le 360°$). Round to the nearest degree.

1) $\sin\theta = \frac{\sqrt{3}}{2}$

2) $\sec\theta = 2$

3) $\tan\theta = -0.1944$

4) $\csc\theta = -1.7013$

5) $\sqrt{2}\sin\theta - 1 = 0$

6) $3\cos x = \cos x + 1$

7) $\sin 3x = 0.5$