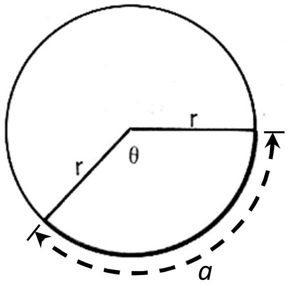
Radian Measure

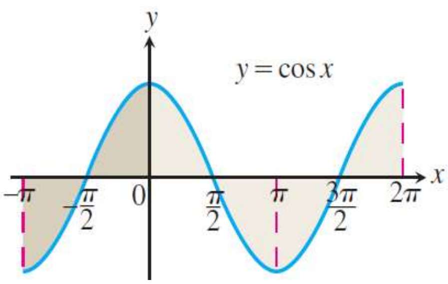


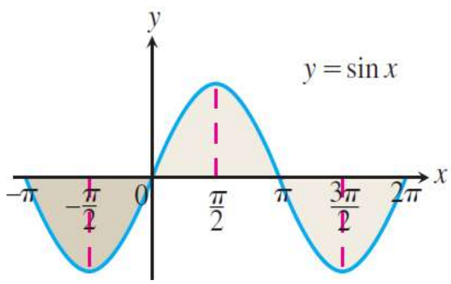
$$\text{number of radians} = \frac{\text{arc length}}{\text{radius}} \quad \text{or} \quad \theta = \frac{a}{r}$$

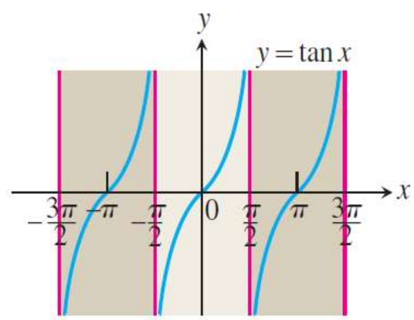
Example 1

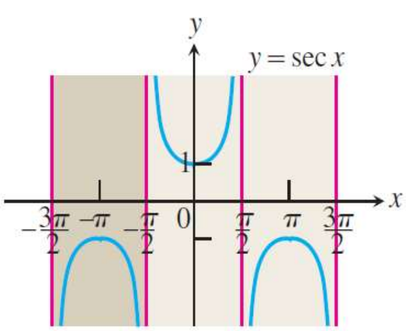
For a radius of 15 and an arc length of 70, determine the central angle, θ, in radian and degree measure.

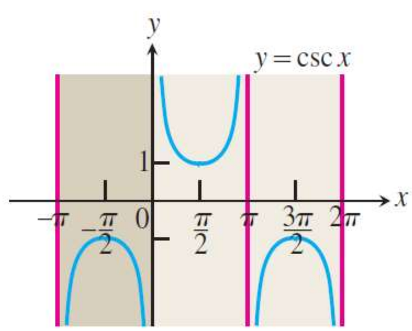
The Graphs of Trigonometric Functions

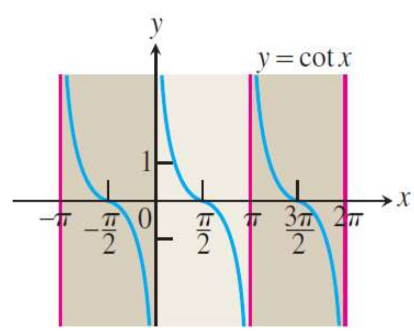












Analysis

State the period of $y = \sin x$ and $y = \cos x$: __________ State the period of $y = \tan x$: __________

State the range of $y = \sin x$ and $y = \cos x$: __________________

State the domain of $y = \sec x$: ________________________________

State the range of $y = \cot x$: __________

List the functions shown above that have even symmetry: _____________________________________

A Brief Investigation for Discussion

Use your calculator to graph the relations described by the following parametric equations (make sure "Simul Graph" is on):

$$x = \cos t,\ y = \sin t \quad \text{and} \quad x = t,\ y = \sin t$$

Experiment with the *Trace* function to compare the values of the two relations and explain how the two graphs are related.

Recall:

$\sin(-x) = -\sin x$	$\cos(-x) = \cos x$	$\tan(-x) = -\tan x$
$\sin\left(\frac{\pi}{2} - x\right) = \cos x$	$\cos\left(\frac{\pi}{2} - x\right) = \sin x$	$\tan\left(\frac{\pi}{2} - x\right) = \cot x$

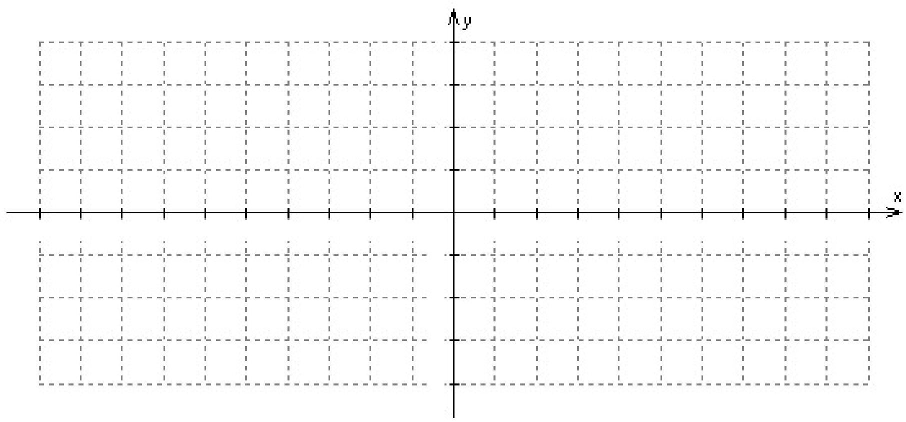
Example 2

Sketch the graph of $y = 3\cos\left(\frac{1}{2}x + \frac{\pi}{2}\right) - 1$ on the interval $-4\pi \le x \le 4\pi$ and state the following properties. Verify your sketch using your calculator.

Amplitude: ____________________

Period: _________________________

Phase Shift: ____________________



Example 3

Sound from a tuning fork is created by pressure waves in the air. The following table shows the pressure displacement versus time (in seconds) of a note produced from a tuning fork.

Time	Pressure	Time	Pressure	Time	Pressure
0.00091	−0.080	0.00271	−0.141	0.00453	0.749
0.00108	0.200	0.00289	−0.309	0.00471	0.581
0.00125	0.480	0.00307	−0.348	0.00489	0.346
0.00144	0.693	0.00325	−0.248	0.00507	0.077
0.00162	0.816	0.00344	−0.041	0.00525	−0.164
0.00180	0.844	0.00362	0.217	0.00543	−0.320
0.00198	0.771	0.00379	0.480	0.00562	−0.354
0.00216	0.603	0.00398	0.681	0.00579	−0.248
0.00234	0.368	0.00416	0.810	0.00598	−0.035
0.00253	0.099	0.00435	0.827		

a) Perform a regression analysis to find an equation that models the data shown above.

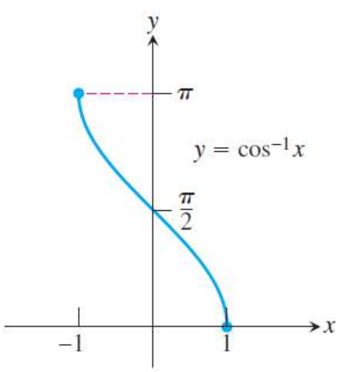
Equation: ____

b) The **frequency** of a sound wave is measured in cycles per second, or hertz (1 Hz = 1 cycle per second). Notice that the frequency is the reciprocal of the wave's period. Estimate the frequency of the note produced by the tuning fork. To what musical note does this frequency correspond?

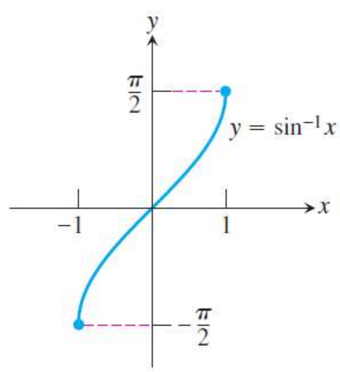
__**Inverse Trigonometric Functions**__

The three primary trigonometric functions and their reciprocal functions are not one-to-one. Therefore, the inverses of these functions will not be functions. If we restrict the domain of the original six functions, however, we can obtain inverses that are indeed functions. Doing so gives us the **inverse trigonometric functions**, as shown below.

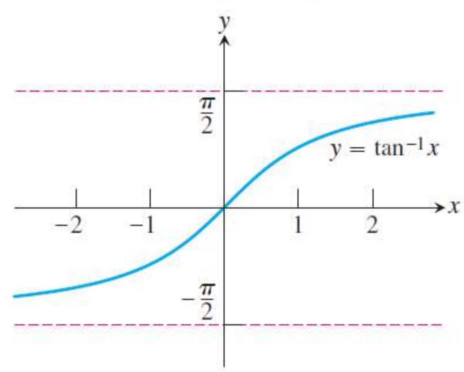
Domain: $-1 \le x \le 1$
Range: $0 \le y \le \pi$



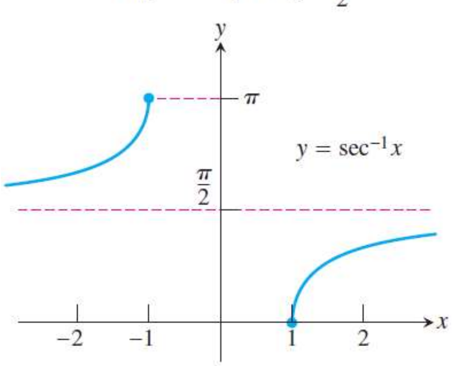
Domain: $-1 \le x \le 1$
Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



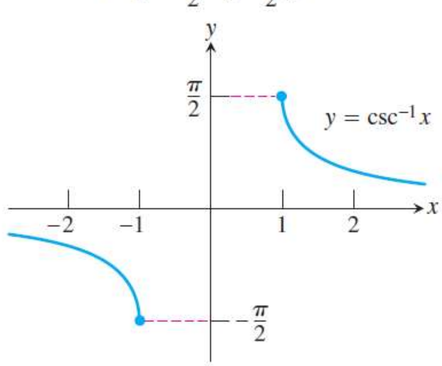
Domain: $-\infty < x < \infty$
Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$



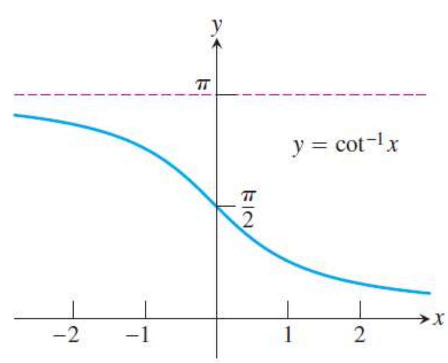
Domain: $x \le -1$ or $x \ge 1$
Range: $0 \le y \le \pi, y \ne \frac{\pi}{2}$



Domain: $x \le -1$ or $x \ge 1$
Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}, y \ne 0$



Domain: $-\infty < x < \infty$
Range: $0 < y < \pi$



Note that the inverse sine of *x* is the angle whose sine is *x*. It is written as $\sin^{-1} x$ or $\arcsin x$. Either notation can be read "the inverse sine of *x*" or "arcsine of *x*." The inverses for the other trigonometric functions are denoted and read in the same manner.

Solving Trigonometric Equations

As we have seen in the past, the inverse trigonometric functions are helpful when solving trigonometric equations.

Example 4

Solve the following equations on the given domains.

a) $\cos x = -0.3 \qquad -2\pi \le x \le 2\pi$

b) $\csc x = 1.7 \qquad -\infty < x < \infty$