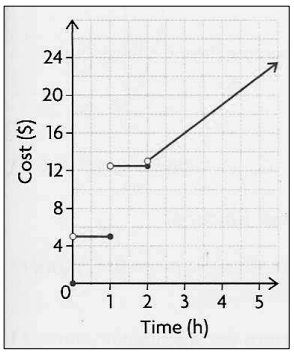
Piecewise Functions

Introduction

A city parking lot uses the following rules to calculate parking fees:

- A flat rate of $5.00 for any amount of time up to and including the first hour.
- A flat rate of $12.50 for any amount of time over 1 hour and up to and including 2 hours.
- A flat rate of $13 plus $3.00 per hour for each hour after 2 hours.

What do you think a graph of cost versus time would look like for this situation?



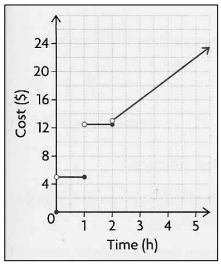
- A flat rate of $5.00 for any amount of time up to and including the first hour.
- A flat rate of $12.50 for any amount of time over 1 hour and up to and including 2 hours.
- A flat rate of $13 plus $3.00 per hour for each hour after 2 hours.

NOTES:
- The domain is $\{x \in \Re \mid x \geq 0\}$.
- The function is linear over the domain.
- The function is discontinuous at $x = 0$, 1, and 2.
- Notice the use of solid and open dots to display whether or not the value is included.

What is a piecewise function?

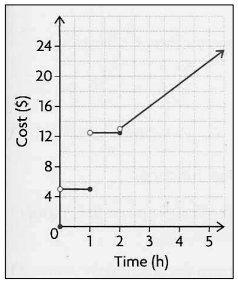
A **piecewise function** is a function that is defined by using different rules on different intervals.

Example $f(x) = \begin{cases} x^2, & x \leq 0 \\ x+2, & x > 0 \end{cases}$



The graph of a **piecewise function** consists of several *pieces*. Each of these pieces is itself a function.

Can we use function notation to represent the parking situation from earlier?



- A flat rate of $5.00 for any amount of time up to and including the first hour.
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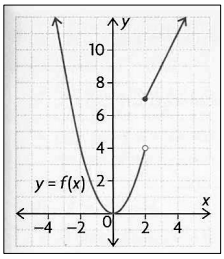
$$f(x) = \begin{cases} 0, & x = 0 \\ 5, & 0 < x \leq 1 \\ 12.50, & 1 < x \leq 2 \\ 3x+7, & x > 2 \end{cases}$$

Some examples…

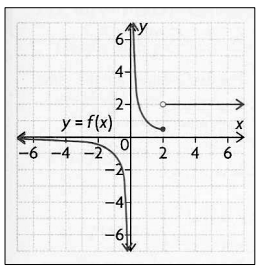
1) Sketch the graph of the following piecewise function.

$$f(x) = \begin{cases} x^2, & x < 2 \\ 2x+3, & x \geq 2 \end{cases}$$



NOTE: $f(x)$ is discontinuous at $x = 2$.

2) Determine the algebraic representation of the following piecewise function.



$$f(x) = \begin{cases} \dfrac{1}{x}, & x \leq 2 \\ 2, & x > 2 \end{cases}$$

How about one more example?

Complete the following on a separate page…

Is the following function continuous at the points where it is pieced together?

$$f(x) = \begin{cases} x+1, & x \leq 0 \\ 2x+1, & 0 < x < 3 \\ 4-x^2, & x \geq 3 \end{cases}$$