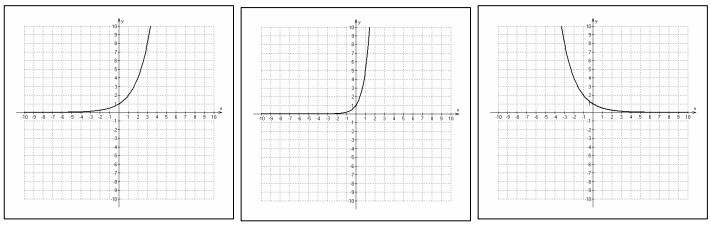
The Logarithmic Function

A little review…

Recall the appearance of the graph of an exponential function:

$y = 2^x$ $y = 3^x$ $y = \left(\frac{1}{2}\right)^x$

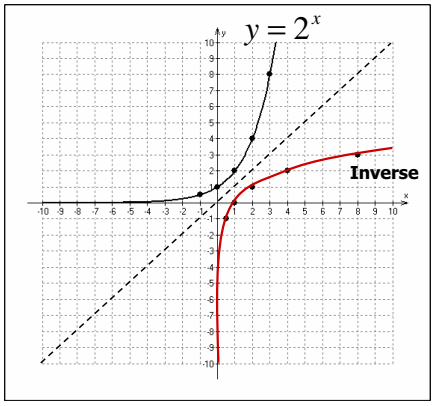


The Inverse of an Exponential Function

Consider the exponential function $y = 2^x$.

We can quickly sketch this function's graph.

We can also sketch the inverse of $y = 2^x$ by reflecting the graph of $y = 2^x$ in the line $y = x$ (switching the x and y coordinates).

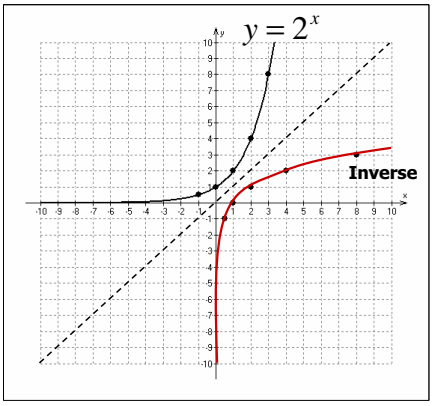


Is the inverse a function?

YES!

Notice that the graph of $y = 2^x$ has a horizontal asymptote along the x-axis and its inverse has a vertical asymptote along the y-axis.

Also, $y = 2^x$ has a y-intercept at 1 and its inverse has an x-intercept at 1.



Introducing the Logarithmic Function

Recall that the inverse of a function is a rule that "undoes" or "reverses" the effect of the function.

Now, with the exponential function $y = 2^x$, the *input* is the exponent and the *output* is the base 2 raised to that exponent.

We know that the inverse of $y = 2^x$ has the equation $x = 2^y$. Notice that for the inverse, the output y is the exponent.

For $x = 2^y$, we can write y as a function of x using the *logarithmic function*, $y = \log_2 x$.

Some important points…

The expression $\log_a x$ means "the **exponent** that must be applied to base a to obtain the value of x."

For a function $y = a^x$, the equation $x = a^y$ is called the *exponential form* of the inverse and the equation $y = \log_a x$ is called the *logarithmic form* of the inverse.

$\log_a x$ is read "the logarithm of x to the base a" or simply "log x base a."

Some examples…

1) Write the inverse of $f(x) = 4^x$ in logarithmic form.

$f(x) = 4^x$

$y = 4^x$

For the inverse:

$x = 4^y$

$y = \log_4 x$

Note that it is not necessary to write the intermediate steps!

$\therefore$ the inverse is $f^{-1}(x) = \log_4 x.$

2) Write $y = \log_3 x$ in exponential form.

$y = \log_3 x$

$3^y = x$

$x = 3^y$

Think: *y* is the exponent that I need to apply to 3 in order to get *x*.

3) Evaluate each of the following.

a) $\log_2 8$

$= 3$

What exponent do I need to apply to a base of 2 to get 8?

b) $\log_3 81$

$= 4$

c) $\log_7 1$

$= 0$

d) $\log_{\frac{1}{2}} \frac{1}{32}$

$= 5$

Question for Discussion

How could we sketch the graph of

$$y = \log_3 x ?$$