Inverse Relations

Review

Recall that the inverse of a function f is the function that "undoes" f.

The inverse of a function f is denoted f^{-1}.

Examples

The inverse of the function $f(x)=x-7$ is $f^{-1}(x)=x+7$.

The inverse of the function $f(x)=4x+3$ is $f^{-1}(x)=\frac{x-3}{4}$.

Since the inverse of a function undoes (or reverses) the function, recall that the domain and range of a function and its inverse are interchanged.

Therefore, we can find the inverse of a function by interchanging the function's *x* and *y* coordinates.

Interpreting Inverse Relations Graphically

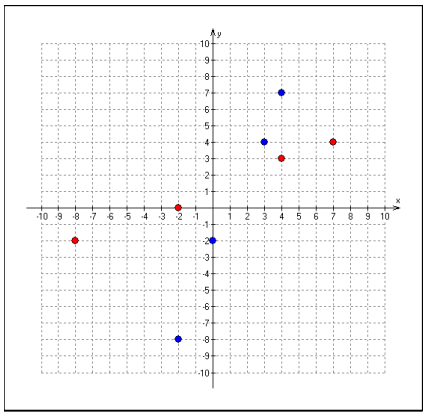
Investigation:

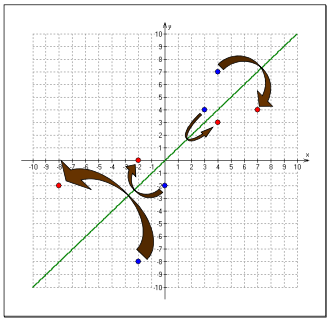
Find the inverse, f^{-1}, of the function f, whose ordered pairs are {(-2, -8), (0, -2), (3, 4), (4, 7)}.

{(-8, -2), (-2, 0), (4, 3), (7, 4)}

f : {(-2, -8), (0, -2), (3, 4), (4, 7)} f^{-1} :{(-8, -2), (-2, 0), (4, 3), (7, 4)}

Graph f and f^{-1}.





How are the graphs of f and f^{-1} related?

The graph of f^{-1} is the reflection of the graph of f in the line $y=x$.

Why does this relationship make sense?

For the inverse of a function, the *x* and *y* coordinates of each point in the original function are interchanged (the domain and range are interchanged).

Find the inverse of the function $f(x)=\sqrt{2x-1}$.

$$f(x)=\sqrt{2x-1}$$
$$y=\sqrt{2x-1}$$

For inverse:

$$x=\sqrt{2y-1}$$
$$x^2=2y-1$$
$$x^2+1=2y$$
$$\frac{x^2+1}{2}=y$$
$$y=\frac{x^2+1}{2}$$

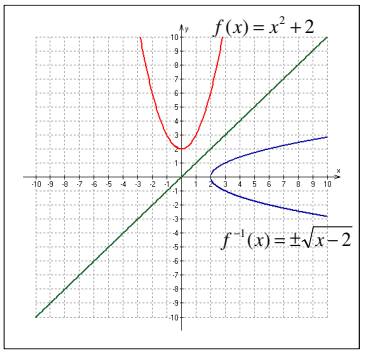
$$\therefore\ f^{-1}(x)=\frac{x^2+1}{2},\ x\geq 0$$

One more example…

Find the inverse of $f(x)=x^2+2$. Sketch the graph of f and f^{-1}.

$$f(x)=x^2+2$$
$$y=x^2+2$$

For inverse:

$$x=y^2+2$$
$$x-2=y^2$$
$$y^2=x-2$$
$$y=\pm\sqrt{x-2}$$

$$\therefore\ f^{-1}(x)=\pm\sqrt{x-2}$$