MHF4U1 – RATES OF CHANGE

SELF-ASSESSMENT QUIZ

1) Determine the average rate of change of the function $f(x)=3x^2-2x+\sqrt{x}$ on the interval $4 \le x \le 9$.

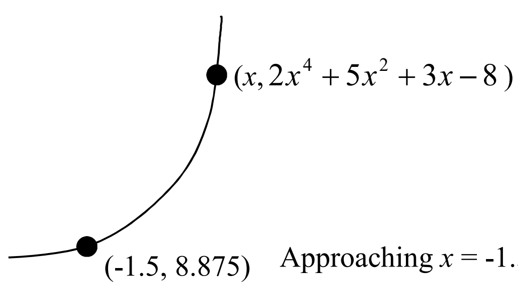
$$\begin{aligned}\text{AROC} &= \frac{\Delta f(x)}{\Delta x}\\ &= \frac{f(9)-f(4)}{9-4}\\ &= \frac{228-42}{5}\\ &= \frac{186}{5}\\ &= 37.2\end{aligned}$$

$\therefore$ the average rate of change is 37.2.

2) Consider the function $g(x)=2x^4+5x^2+3x-8$.

a) Determine the instantaneous rate of change of the function $g(x)$ at the point where x = -1.5.

$$g(-1.5)=8.875$$



$$\begin{aligned}m_{\text{secant}} &= \frac{\Delta g(x)}{\Delta x}\\ &= \frac{2x^4+5x^2+3x-8-8.875}{x-(-1.5)}\\ &= \frac{2x^4+5x^2+3x-16.875}{x+1.5}\end{aligned}$$

Approaching x = -1.5 from the left:

x	m_{secant}
-1.51	-39.32120199
-1.501	-39.032012
-1.5001	-39.0032
-1.50001	-39.00032

Approaching x = -1.5 from the right:

x	m_{secant}
-1.49	-38.681198
-1.499	-38.968012
-1.4999	-38.9968
-1.49999	-38.99968

$\therefore$ the instantaneous rate of change is approximately -39.

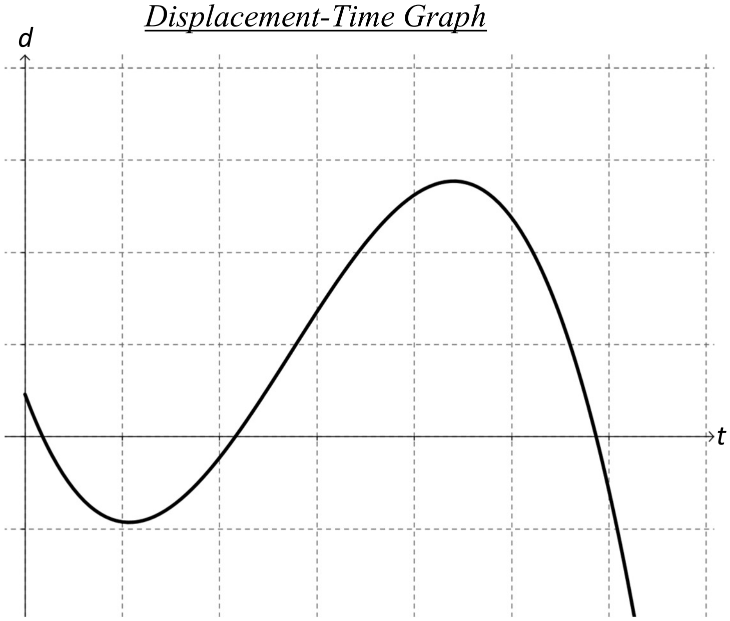
b) Determine the equation of the tangent line to $g(x)$ at the point where x = -1.5.

$$\begin{aligned}y &= mx+b\\ y &= -39x+b\\ 8.875 &= -39(-1.5)+b\\ b &= -49.625\end{aligned}$$

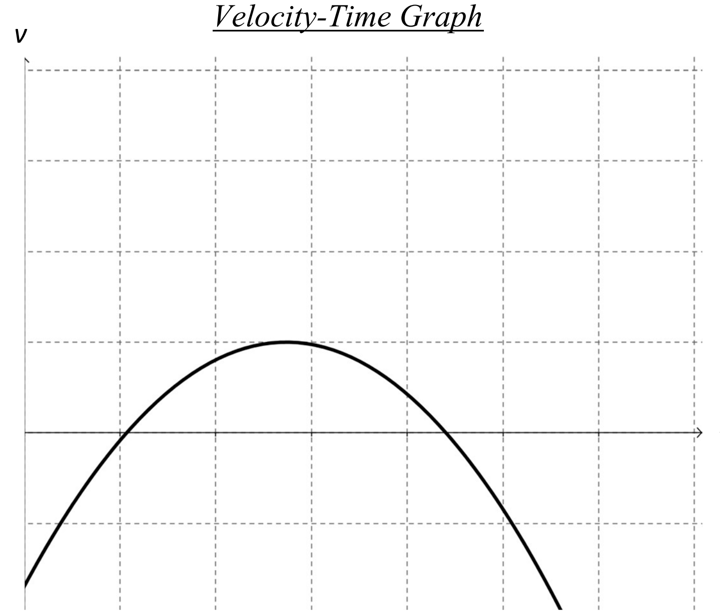
$\therefore$ the equation of the tangent is $y=-39x-49.625$

3) A displacement-time graph is shown below. Sketch the corresponding velocity-time graph.

Displacement-Time Graph



Velocity-Time Graph



4) Does the graph of $y = -\frac{1}{x}$ have any points at which the instantaneous rate of change is negative? Explain.

As shown on the right, the graph of $y = -\frac{1}{x}$ is a vertical reflection of the graph of $y = \frac{1}{x}$. Since this function never decreases, it never has a negative tangent slope. Therefore, the function $y = -\frac{1}{x}$ never has a negative instantaneous rate of change.