Transformations of Logarithmic Functions

The easiest way to deal with graphing logarithmic functions is to remember that they are the inverses of exponential functions.

So, in your mind, reverse the roles of *x* and *y* for the exponential function and the graph of the logarithmic function comes easy!

For example, consider the graph of $y = \log_{10} x$, which is the inverse of $y = 10^x$.

$y = 10^x$	$y = \log_{10} x$
y-intercept of 1	*x*-intercept of 1
$f(x) \to 0$ as $x \to -\infty$ (*x*-axis is an asymptote)	$f(x) \to -\infty$ as $x \to 0$ (*y*-axis is an asymptote)
As $x \to \infty$, $f(x) \to \infty$ (quickly)	As $f(x) \to \infty$, $x \to \infty$ (quickly)

Also, the coordinates of $y = \log_{10} x$ are the "reverse" of the coordinates of $y = 10^x$.

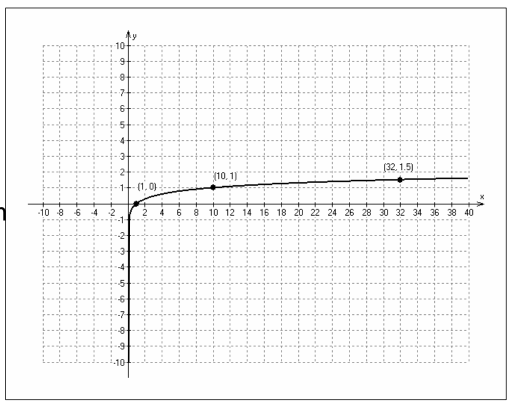
As we have done with several other functions, we can apply transformations to the graphs of logarithmic functions.

Example: Sketch the graph of $y = -2\log_{10}(x-4)$. State the domain and range of the resulting function.

Start by sketching the graph of $y = \log_{10} x$.

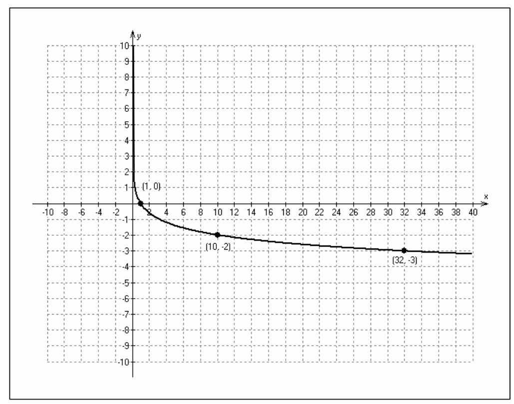
Remember that this can be written as $10^y = x$.

Notice that the point (32, 1.5) is an approximation.



$y = -2\log_{10}(x-4)$

Now, apply a vertical reflection (in the *x*-axis) and a vertical stretch of factor 2.



$y = -2\log_{10}(x-4)$

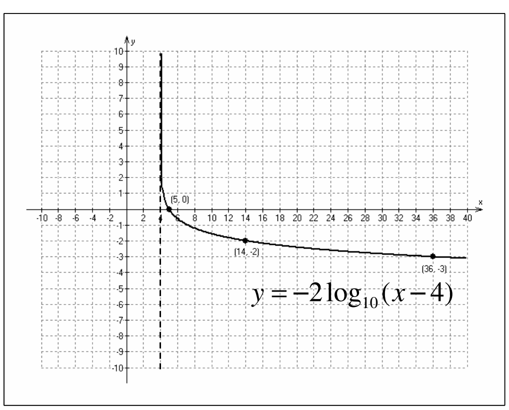
Finally, apply a horizontal shift of 4 units to the right.

DON'T FORGET TO SHIFT THE ASYMPTOTE!

Note: You may choose to apply all of the transformations in one step, but be careful!

Domain: $\{x \in \Re \mid x > 4\}$

Range: $\{y \in \Re\}$



One more example…

The graph logarithmic function $y = \log_5 x$ has been vertically compressed by a factor of 2/3, horizontally stretched by a factor of 4 and reflected in the *y*-axis. It has also been horizontally translated so that the vertical asymptote is *x* = -2 and then vertically translated 3 units down. Write an equation for the transformed graph and state its domain and range.

$$y = \frac{2}{3}\log_5\left[-\frac{1}{4}(x+2)\right] - 3$$

Domain: $\{x \in \Re \mid x < -2\}$

Range: $\{y \in \Re\}$