APPLICATIONS OF THE DOT PRODUCT AND THE CROSS PRODUCT

The word *work* is commonly applied to any activity that requires physical exertion or mental effort. In physics, however, the word *work* has a much narrower meaning.

Work

Work is done whenever a force acting on an object causes a displacement of the object from one position to another.

The work done in moving an object is given by the component of the force in the direction of motion multiplied by the distance traveled. Using vectors, we obtain,

$$\begin{aligned} W &= \vec{F} \bullet \vec{d} \\ &= |\vec{F}|\,|\vec{d}| \cos\theta, \end{aligned}$$

where W is the work done, $\vec{F}$ is the force acting on the object, $\vec{d}$ is the displacement caused by the force and θ is the angle between the force and displacement vectors.

The unit of work is a **newton metre**, also called a **joule** (**J**).

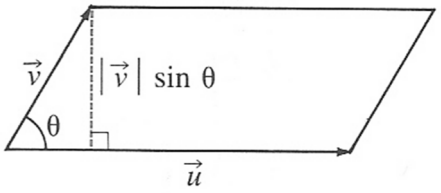
Notice that work is a scalar quantity.

Example

A crate on a ramp is hauled 8 m up the ramp under a constant force of 20 N applied at an angle of 25° to the ramp. Find the work done.

Area of a Parallelogram

The area of a parallelogram is found by multiplying its base and its height. In the diagram on the right, the base of the parallelogram is $|\vec{u}|$ and the height is $|\vec{v}|\sin\theta$.



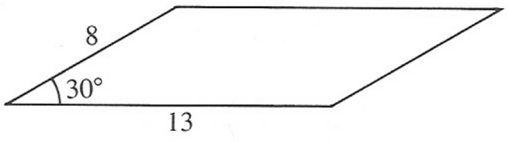
- Therefore, the area of a parallelogram is $|\vec{u}||\vec{v}|\sin\theta$. Does this expression look familiar?!?

Area of a Parallelogram

The area of a parallelogram with adjacent sides $\vec{u}$ and $\vec{v}$ is $|\vec{u}\times\vec{v}|$.

Remember: $\vec{u}$ and $\vec{v}$ must be arranged tail-to-tail

Example
Find the area of the following parallelogram.



Example
The points $A(1, 1, 1)$, $B(-2, 0, -4)$, $C(1, 2, -3)$ and $D(4, 3, 2)$ are the vertices of a parallelogram. Find the area of each of the following figures.

a) *ABCD*

b) Triangle *ABC*

Torque

When a threaded bolt is tightened by applying a force to a wrench, the measure of the *turning effect* of the force on the bolt is called **torque**.

- The magnitude of the turning effect (torque) depends on two factors.

    1) The distance between the bolt and the point where the force is applied.

    2) The magnitude of the force directed perpendicular to the wrench.

Magnitude of Torque

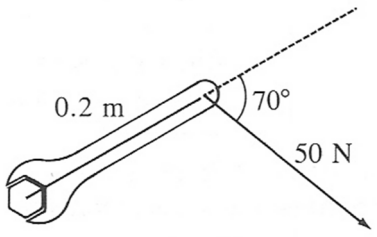
The magnitude of the **torque** caused by a force is defined as

$$\begin{aligned} \left|\vec{T}\right| &= \left|\vec{r} \times \vec{F}\right| \\ &= \left|\vec{r}\right|\left|\vec{F}\right| \sin\theta \end{aligned}$$

where $\vec{T}$ is the torque, $\vec{r}$ is the *radius vector* from the centre of rotation to the point where the force is applied, $\vec{F}$ is the applied force and θ is the angle between $\vec{r}$ and $\vec{F}$ (when arranged tail-to-tail).

- Torque is measured in units of newton metres (N•m).

Example
A bolt is tightened by applying a 50 N force to a 0.2 m wrench as shown in the following diagram. Calculate the magnitude of the torque.



In what direction does the torque act?

What angle would give the maximum torque? Why?