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NAME: _____

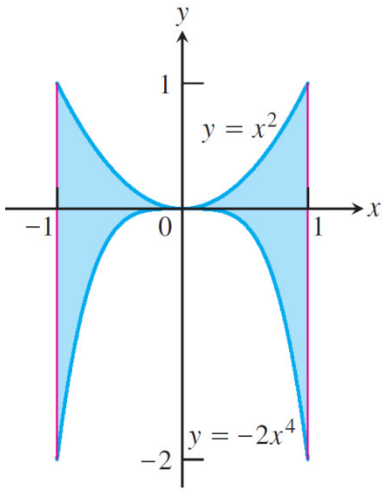
KNOW / 12	APP / 12	INQ / 12	COMM / 6

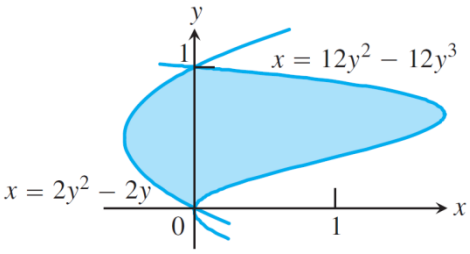
MCV4UP - UNIT 9 – APPLICATIONS OF DEFINITE INTEGRALS

TEST

GIVE ALL ANSWERS IN EXACT FORM AND SHOW ALL WORK, UNLESS STATED OTHERWISE.

1) Determine the area of the shaded region in the diagram to the right**.** (*K – 4 marks*)





2) **Using a calculator to integrate**, determine the **exact** area of the shaded region in the diagram on the left. (*K – 4 marks*)

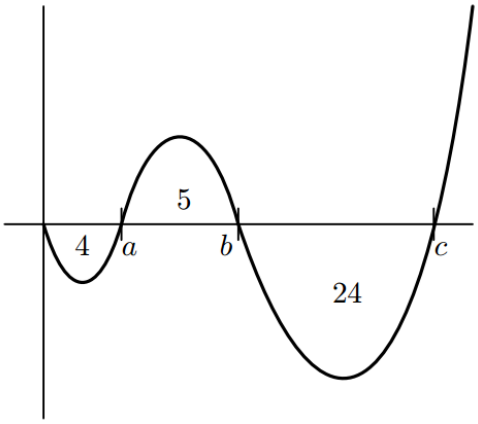
3) **Using a calculator to integrate**, determine the **exact** area of the region enclosed by the following curves. (*K – 4 marks*)

$$y = x^3 - x\,, \qquad y = 3x$$

4) Determine the **exact** area of the region in the first quadrant enclosed by the curves $y = x^2$, $x + y = 2$ and the x-axis. (*A – 6 marks*)

5) Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ litres per minute, where $0 \le t \le 50$. Determine the amount of water that flows from the tank during the first 10 minutes. (*A – 2 marks*)

6) A particle is moving along the x-axis. Its initial position is $s(0) = 15$ metres. The graph on the right shows the particle's velocity, $v(t)$, where velocity is measured in m/s. The numbers on the graph are the areas of the enclosed regions.



a) Determine the particle's net displacement from $t = 0$ to $t = c$. (*A – 1 mark*)

Net displacement = _____

b) Determine the particle's position at each of the following times. (*A – 2 marks*)

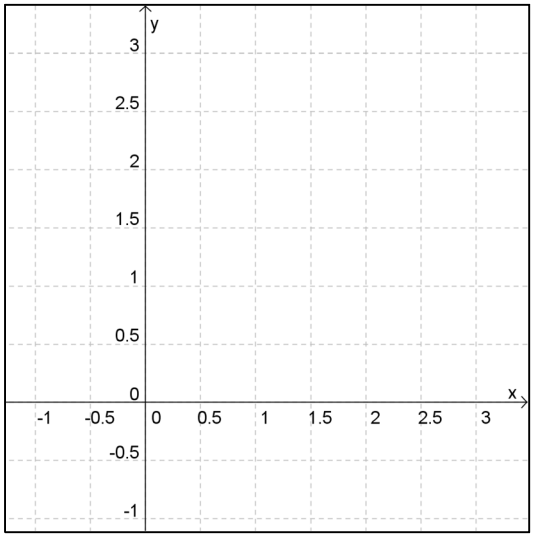
i) At $t = a$, position = _____ ii) At $t = b$, position = _____

c) True or false (*check one*): (*A – 1 mark*)

$\int_0^c v(t)dt$ gives the total distance travelled from $t = 0$ to $t = c$. ❒ True ❒ False

7) Consider the region enclosed by the following curves:

$$y = e^{-x}, \quad y = 1, \quad x = 2$$



a) Use the axes on the right and shading to illustrate the region described above. (*I – 1 mark*)

b) Using the **washer method** (cross sections), determine the volume of the solid generated by revolving the given region about the ***x*-axis**. Round your answer to the **nearest tenth**. (*I – 4 marks*)

c) Using the **method of cylindrical shells**, determine, **to the nearest tenth**, the volume of the solid generated by revolving the given region about the ***y*-axis**. **You may use a calculator to integrate**. (*I – 3 marks*)

d) Set up, but do not evaluate, an integral to determine the volume of the solid generated by revolving the given region about each of the following lines. (*I – 4 marks*)

i) The line $y = 1$.

ii) The line $y = 2$

8) The base of a solid is the region between the curve $y = x^3$ and the ***y*-axis** for $0 \le x \le 2$. Each cross section is a right triangle for which the base is **perpendicular to the *y*-axis** (the base and height form the right angle at the *y*-axis). The height of each triangular cross section is equal to its base. Determine the volume of the solid to the **nearest tenth**. **You may use your calculator's integration function.** (*C – 4 marks*)

9) Two runners, *A* and *B*, run on a straight racetrack for 10 seconds. The graph on the right shows the velocity for runner *A* in metres per second. The velocity, in metres per second, for runner *B* is given by $v(t) = \dfrac{24t}{2t+3}$. Determine how much more distance runner *A* has travelled than runner *B* at the end of the 10 second interval. Round your answer to the **nearest tenth**. **You may use your calculator's integration function, if needed.** (*C – 2 marks*)