

Exploring Operations with Functions



Suppose we had two functions, $f(x)$ and $g(x)$.

Now, let's think about what $f(x) + g(x)$ means.

Well, for a moment, let's consider the case where $x = 2$. If we want to find $f(2) + g(2)$, we could simply evaluate both functions at $x = 2$ and then add the results.

A few things to think about with $f(2) + g(2)$...

- * Notice that it is the two output values (y-values) that we're actually adding here!
- * Notice that we are adding the output values that correspond to a single input value. That is, if we substitute $x = 2$ into $f(x)$, we must substitute $x = 2$ into $g(x)$.
- * Notice that we can only perform this addition if 2 is in the domain of both $f(x)$ and $g(x)$.



Let's get to the point...

- * When we add two functions, we add the two output values (y-values) for every valid input value.
- * We can add the two functions only where their domains overlap.
- * The same ideas apply to the subtraction and multiplication of two functions.



Some examples...

- 1) If $f = \{(1, 2), (2, -3), (3, 4), (5, 8), (6, -7)\}$ and $g = \{(1, 2), (2, 5), (3, -6), (4, -8), (5, 3)\}$, find the following:
 - a) $f + g = \{(1, 4), (2, 2), (3, -2), (5, 11)\}$
 - b) $f - g = \{(1, 0), (2, -8), (3, 10), (5, 5)\}$
 - c) $g - f = \{(1, 0), (2, 8), (3, -10), (5, -5)\}$
 - d) $fg = \{(1, 4), (2, -15), (3, -24), (5, 24)\}$

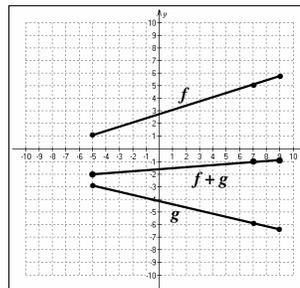
Question for Discussion: How does the domain of each of the results above compare to the domains of the original two functions?

2) If $f(x) = 7x + 5$ and $g(x) = 9x - 12$, find an equation for the following:

- a)
$$\begin{aligned} f(x) - g(x) &= (7x + 5) - (9x - 12) \\ &= 7x + 5 - 9x + 12 \\ &= -2x + 17 \end{aligned}$$
- b)
$$\begin{aligned} f(x) \times g(x) &= (7x + 5)(9x - 12) \\ &= 63x^2 - 84x + 45x - 60 \\ &= 63x^2 - 39x - 60 \end{aligned}$$



3) Use the graphs of functions f and g to sketch the graph of $f + g$.



Questions for Discussion:

- * Would the graph of $f - g$ look the same as the graph of $g - f$?
- * What result would you expect to obtain if you were to sketch the graph of fg ?

