THE QUOTIENT RULE

Does the title remind you of anything? The product rule! The product rule states the following:

The derivative of the first times the second plus the first times the derivative of the second.

In another form, the product rule states $(fg)' = f'g + fg'$.

The product rule was convenient for functions that were the product of the two functions of which we knew how to take the derivative, but were too difficult (or impossible) to reduce to a regular polynomial function.

Now we will look at functions that are the quotient of two functions of which we know how to take the derivative (rational functions). For these functions, we can differentiate using the **quoutient rule**, which states:

The Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2} \quad \text{or} \quad \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{\left[\frac{d}{dx}f(x)\right]g(x) - \left[\frac{d}{dx}g(x)\right]f(x)}{\left[g(x)\right]^2}$$

The derivative of the top times the bottom, minus the derivative of the bottom times the top, all over the bottom squared.

Examples: Differentiate each of the following functions

1) $y = \dfrac{2x+5}{3x-1}$

2) $y = \dfrac{x^3-3}{1+4x^2}$

3) $y = \dfrac{x^2}{(x+2)(x-3)}$