

Sketch the graph of the function $f(x) = \frac{x^2 - 1}{x^3}$.

$$f(x) = \frac{(x+1)(x-1)}{x^3}$$

Domain

$$\{x \in \mathbb{R} \mid x \neq 0\}$$

Intercepts

x-intercept:

$$0 = \frac{(x+1)(x-1)}{x^3}$$

$$x = -1, 1$$

y-intercept:

$$f(0) = \frac{0^2 - 1}{0^3}$$

$$= \frac{-1}{0}$$

Undefined (no y-intercept)

Asymptotes

Vertical: $x = 0$

Horizontal: $y = 0$

Oblique: none

Intervals of Increase/Decrease

$$f(x) = \frac{x^2 - 1}{x^3}$$

$$f'(x) = \frac{2x(x^3) - 3x^2(x^2 - 1)}{(x^3)^2}$$

$$= \frac{2x^4 - 3x^4 + 3x^2}{x^6}$$

$$= \frac{-x^4 + 3x^2}{x^6}$$

$$= \frac{-x^2(x^2 - 3)}{x^6}$$

$$= \frac{-(x^2 - 3)}{x^4}$$

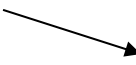
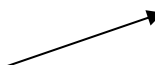
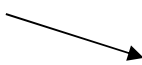
For critical numbers,

$$0 = \frac{-(x^2 - 3)}{x^4}$$

$$x = \pm\sqrt{3}$$

$$x = 0$$

Also, $f'(x)$ is undefined for $x = 0$.

	$x < -\sqrt{3}$	$-\sqrt{3} < x < 0$	$0 < x < \sqrt{3}$	$x > \sqrt{3}$
Sign of $f'(x)$	-	+	+	-
Increase/Decrease for $f(x)$				

Maximum and Minimum Points

Local minimum at $(-\sqrt{3}, -0.38)$.

Local maximum at $(\sqrt{3}, 0.38)$

Concavity

$$f'(x) = \frac{3-x^2}{x^4}$$

$$\begin{aligned} f''(x) &= \frac{-2x(x^4) - 4x^3(3-x^2)}{(x^4)^2} \\ &= \frac{-2x^5 - 12x^3 + 4x^5}{x^8} \\ &= \frac{2x^5 - 12x^3}{x^8} \\ &= \frac{2x^3(x^2 - 6)}{x^8} \\ &= \frac{2(x^2 - 6)}{x^5} \end{aligned}$$

For possible inflection points,

$$0 = \frac{2(x^2 - 6)}{x^5}$$

$$x = \pm\sqrt{6}$$

Also, $f''(x)$ is undefined for $x = 0$.

	$x < -\sqrt{6}$	$-\sqrt{6} < x < 0$	$0 < x < \sqrt{6}$	$x > \sqrt{6}$
Sign of $f''(x)$	-	+	-	+
Concavity of $f(x)$	Down	Up	Down	Up

Inflection Points

Inflection points at $(-\sqrt{6}, -0.34)$ and $(\sqrt{6}, 0.34)$

