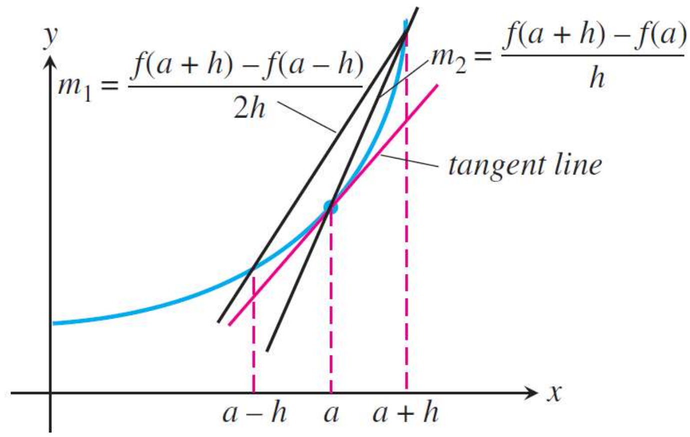
The Numerical Derivative

We know that for very small values of *h*, the difference quotient $\frac{f(a+h)-f(a)}{h}$ gives us a good approximation of the tangent slope at *a*.

Another quotient, however, known as the **symmetric difference quotient** will give an even better approximation.

The Symmetric Difference Quotient

$$\frac{f(a+h)-f(a-h)}{2h}$$



Questions for Discussion

- Does the symmetric difference quotient still represent the slope of a secant line?
- Does $\lim_{h\to 0}\frac{f(a+h)-f(a-h)}{2h}$ still represent the derivative?

Examples

1) Use the symmetric difference quotient with an *h*-value of 0.001 to approximate the derivative of $f(x)=-2x^3$ at *x* = 4.

NOTES:

- When approximating the derivative this way, we are said to be finding the **numerical derivative**.
- Our textbook uses *h* = 0.001 when computing numerical derivatives.

2) Use the *d*/*dx* function on your calculator to find the numerical derivative of $f(x)=\frac{3x^2-5x+1}{x-6}$ at *x* = 3.5. Answer: ______________________

NOTE: Our textbook uses the notation **NDER** to indicate the numerical derivative.

3) Can the numerical derivative be used to calculate the derivative of $f(x)=|x-3|$ at *x* = 3?

4) Use your calculator's numerical derivative function to graph $y=f'(x)$ if $f(x)=\sin x$. Hypothesize the equation of the derivative.

<u>Differentiability and Continuity</u>

If a function is differentiable at $x = a$, must it be continuous at a? __________

Proof:

If a function is continuous at $x = a$, must it be differentiable at a? ___________

Proof:

<u>The Intermediate Value Theorem for Derivatives</u>

Question for discussion: Is $y = 2x$ the derivative of some function?

Not every function can be a derivative. In order for this property to occur, the derivative function must possess the intermediate value property.

THEOREM Intermediate Value Theorem for Derivatives

If a and b are any two points in an interval on which f is differentiable, then f' takes on every value between $f'(a)$ and $f'(b)$.

(The proof of this theorem is beyond the scope of this course!)

The tangent slopes on the original function *f* take on every value between the tangent slope at *a* and the tangent slope at *b*.

Example

Could the *unit step function*, as shown on the right, be the derivative of another function?