

KNOW	/ 12	APP	/ 12	INQ	/ 12	COMM	/ 6
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MCV4UP - UNIT 3 – DERIVATIVES**TEST****GIVE ALL ANSWERS IN EXACT FORM, UNLESS STATED OTHERWISE.****PART A – This section is to be completed without the use of a calculator. Upon completing this section, hand it in to receive the remainder of the test.**

1) Differentiate each of the following functions. Final answers should not contain negative or fraction exponents and should be simplified/factored as much as possible. (*K – 2 marks each*)

a) $f(x) = 5x^4 - 7x + \frac{5}{x^3}$

b) $f(x) = 2\sqrt[4]{16x^3}$

c) $f(x) = (2x^3 - 3x^2 + x - 5)(x^2 + 1)$

d) $f(x) = \frac{\tan x}{8x}$

e) $f(x) = -\cos^2 x$

f) $f(x) = \frac{x^2 + 9}{3\csc x}$

**PART B – This section may be completed
with the use of a calculator.**

NAME: _____

- 2) The position, from a fixed point, of an object moving along a straight line is given by

$s(t) = \frac{4}{3}t^3 - 10t^2 + 24t + 6$, where $s(t)$ is the object's position, in metres, after t seconds.

a) Find the object's velocity at 3.5 seconds. (*A – 2 marks*)

b) Find the object's initial acceleration. (*A – 2 marks*)

c) Determine when the object is stationary. (*A – 2 marks*)

d) Determine the farthest distance that the object achieves **from its starting position** within the first 5 seconds. (*A – 2 marks*)

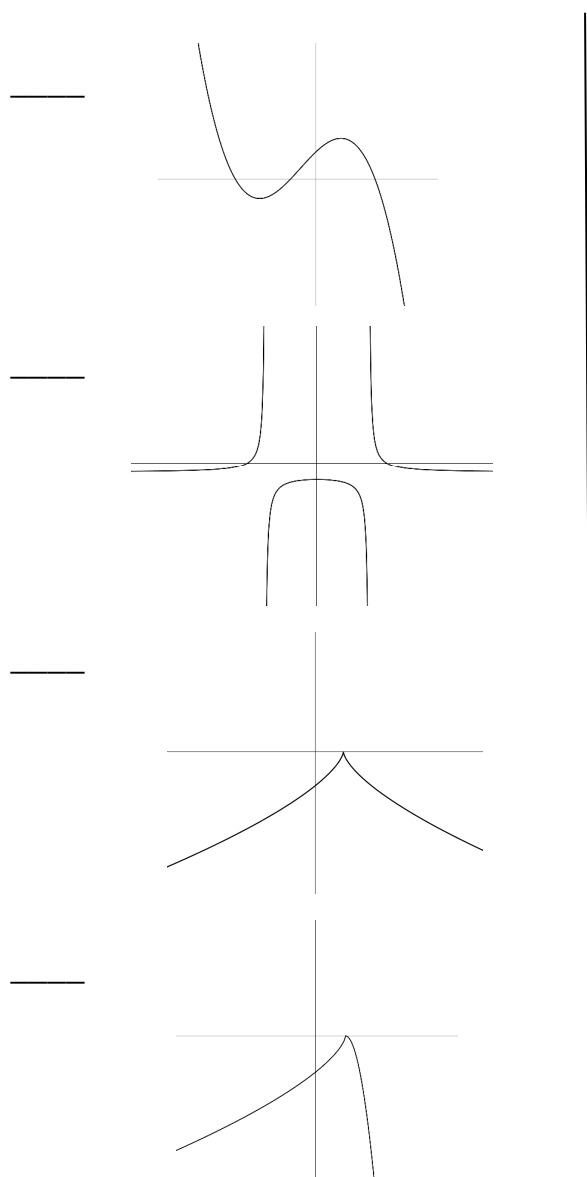
- 3) Company data suggests that the total dollar cost of a certain flight is approximately

$C(x) = 0.0005x^3 - 0.38x^2 + 120x$, where x is the number of passengers.

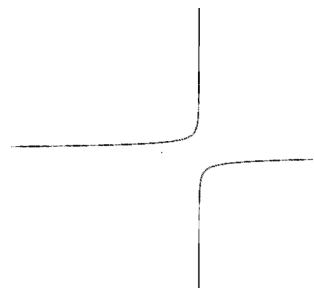
a) Determine the marginal cost when the number of passengers is 150. (*A – 2 marks*)

b) Is it more expensive to add a passenger when $x = 150$ or when $x = 200$? (*A – 2 marks*)

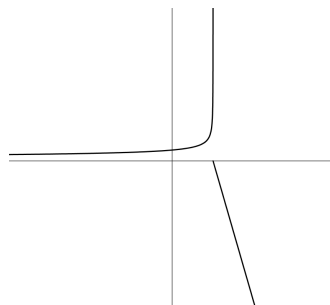
- 4) For each of the following graphs on the left, write the letter of the corresponding derivative graph on the right. (1 – 2 marks)



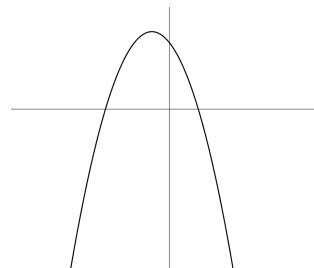
a)



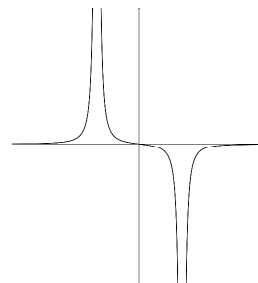
b)



c)



d)

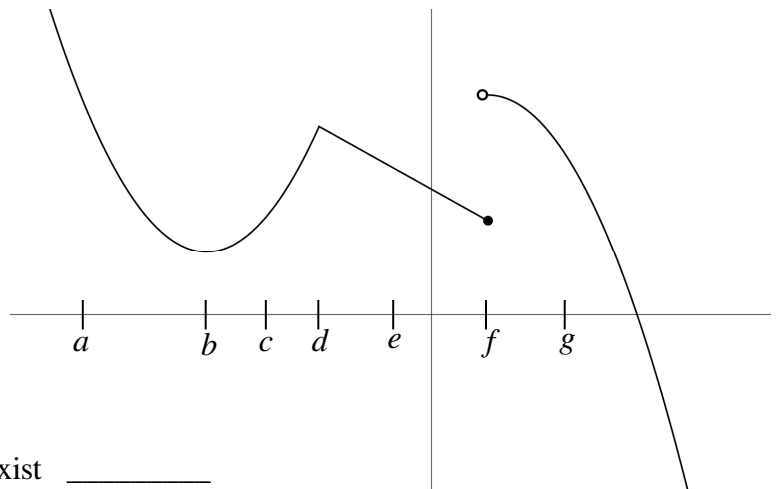


- 5) Using calculus, show algebraically that the tangents to the curve $f(x) = -2x^2 + 12x - 14$ never pass through the point $(4, -5)$. (1 – 3 marks)

- 6) Determine the values of x at which the tangent to the curve $f(x) = \frac{2}{3}x^3 + \frac{7}{2}x^2 - x + \frac{5}{3}$ is perpendicular to the line $y = -\frac{1}{3}x + 7$. (1 – 3 marks)

- 7) For the graph of $y = f(x)$ shown on the right, use the given letters to state a possible x -value where each of the following conditions is met.
(1 – 4 marks)

- a) $f'(x) > 0$ _____
- b) $f'(x) = 0$ _____
- c) $f'(x)$ is constant _____
- d) $\lim_{x \rightarrow \text{letter}} f(x)$ exists, but $f'(x)$ does not exist _____



- 8) Use the definition of the derivative (first principles) to find the derivative of $f(x) = x^3 + 8x^2$.
($C - 2$ marks)

- 9) A disgruntled calculus student claimed that the function shown on the right is not differentiable at $x = 4$. Is the student's claim correct? Explain. ($C - 2$ marks).

$$f(x) = \begin{cases} -\frac{3}{8}x^2 + \frac{7}{2}x - 7, & x \leq 4 \\ 2\sqrt{x} - 3, & x > 4 \end{cases}$$

- 10) Prove the Constant Multiple Rule of Differentiation. That is, prove that if $g(x) = cf(x)$, where $f(x)$ is differentiable, then $g'(x) = cf'(x)$. ($C - 2$ marks)