Sketch the graph of the function $f(x)=\dfrac{x^2-1}{x^3}$.

$$f(x)=\frac{(x+1)(x-1)}{x^3}$$

Domain

$\{x\in\Re \mid x\neq 0\}$

Intercepts

x-intercept:

$$0=\frac{(x+1)(x-1)}{x^3}$$

$$x=-1,1$$

y-intercept:

$$f(0)=\frac{0^2-1}{0^3}$$

$$=\frac{-1}{0}$$

Undefined (no *y*-intercept)

Asymptotes

Vertical: $x=0$
Horizontal: $y=0$
Oblique: none

Intervals of Increase/Decrease

$$f(x)=\frac{x^2-1}{x^3}$$

$$f'(x)=\frac{2x(x^3)-3x^2(x^2-1)}{(x^3)^2}$$

$$=\frac{2x^4-3x^4+3x^2}{x^6}$$

$$=\frac{-x^4+3x^2}{x^6}$$

$$=\frac{-x^2(x^2-3)}{x^6}$$

$$=\frac{-(x^2-3)}{x^4}$$

For critical numbers,

$$0=\frac{-(x^2-3)}{x^4}$$

$$x=\pm\sqrt{3}$$

$$x=4$$

Also, $f'(x)$ is undefined for $x=0$.

	$x<-\sqrt{3}$	$-\sqrt{3}<x<0$	$0<x<\sqrt{3}$	$x>\sqrt{3}$
Sign of $f'(x)$	-	+	+	-
Increase/Decrease for $f(x)$	↘	↗	↗	↘

Maximum and Minimum Points

Local minimum at $(-\sqrt{3},-0.38)$.
Local maximum at $(\sqrt{3},0.38)$

Concavity

$$f'(x)=\frac{3-x^2}{x^4}$$

$$f''(x)=\frac{-2x(x^4)-4x^3(3-x^2)}{(x^4)^2}$$

$$=\frac{-2x^5-12x^3+4x^5}{x^8}$$

$$=\frac{2x^5-12x^3}{x^8}$$

$$=\frac{2x^3(x^2-6)}{x^8}$$

$$=\frac{2(x^2-6)}{x^5}$$

For possible inflection points,

$$0=\frac{2(x^2-6)}{x^5}$$

$$x=\pm\sqrt{6}$$

Also, $f''(x)$ is undefined for $x = 0$.

	$x<-\sqrt{6}$	$-\sqrt{6}<x<0$	$0<x<\sqrt{6}$	$x>\sqrt{6}$
Sign of $f''(x)$	-	+	-	+
Concavity of $f(x)$	Down	Up	Down	Up

Inflection Points

Inflection points at $(-\sqrt{6},-0.34)$ and $(\sqrt{6},0.34)$