Sketch the graph of the function $f(x)=\dfrac{x^2-8x+16}{x^2-6x+9}$.

$$f(x)=\frac{(x-4)^2}{(x-3)^2}$$

Domain

$$\{x\in\Re \mid x\neq 3\}$$

Intercepts

x-intercept:

$$0=\frac{(x-4)^2}{(x-3)^2}$$

$$0=(x-4)^2$$

$$x=4$$

y-intercept:

$$f(0)=\frac{(0-4)^2}{(0-3)^2}$$

$$=\frac{16}{9}$$

$$\doteq 1.8$$

Asymptotes

Vertical: $x = 3$
Horizontal: $y = 1$
Oblique: none

Intervals of Increase/Decrease

$$f(x)=\frac{(x-4)^2}{(x-3)^2}$$

$$f'(x)=\frac{2(x-4)(x-3)^2-2(x-3)(x-4)^2}{(x-3)^4}$$

$$=\frac{2(x-4)(x-3)\left[(x-3)-(x-4)\right]}{(x-3)^4}$$

$$=\frac{2(x-4)}{(x-3)^3}$$

For critical numbers,

$$0=\frac{2(x-4)}{(x-3)^3}$$

$$x=4$$

Also, $f'(x)$ is undefined for $x = 3$.

	$x<3$	$3<x<4$	$x>4$
Sign of $f'(x)$	+	-	+
Increase/Decrease for $f(x)$	↗	↘	↗

Maximum and Minimum Points

Local minimum at (4, 0).

Concavity

$$f'(x) = \frac{2(x-4)}{(x-3)^3}$$

$$f''(x) = \frac{2(x-3)^3 - 3(x-3)^2(2)(x-4)}{(x-3)^6}$$

$$= \frac{2(x-3)^2\left[x-3-3(x-4)\right]}{(x-3)^6}$$

$$= \frac{2(-2x+9)}{(x-3)^4}$$

$$= \frac{-2(2x-9)}{(x-3)^4}$$

For possible inflection points,

$$0 = \frac{-2(2x-9)}{(x-3)^4}$$

$x = 4.5$

Also, $f''(x)$ is undefined for $x = 3$.

	$x < 3$	$3 < x < 4.5$	$x > 4.5$
Sign of $f''(x)$	+	+	-
Concavity of $f(x)$	Up	Up	Down

Inflection Points

Inflection point at (4.5, 0.1)