

Sketch the graph of the function $f(x) = \frac{x^2 - 8x + 16}{x^2 - 6x + 9}$.

$$f(x) = \frac{(x-4)^2}{(x-3)^2}$$

Domain

$$\{x \in \mathbb{R} \mid x \neq 3\}$$

Intercepts

x-intercept:

$$0 = \frac{(x-4)^2}{(x-3)^2}$$

$$0 = (x-4)^2$$

$$x = 4$$

y-intercept:

$$f(0) = \frac{(0-4)^2}{(0-3)^2}$$

$$= \frac{16}{9}$$

$$\doteq 1.8$$

Asymptotes

Vertical: $x = 3$

Horizontal: $y = 1$

Oblique: none

Intervals of Increase/Decrease

$$f(x) = \frac{(x-4)^2}{(x-3)^2}$$

$$f'(x) = \frac{2(x-4)(x-3)^2 - 2(x-3)(x-4)^2}{(x-3)^4}$$

$$= \frac{2(x-4)(x-3)[(x-3) - (x-4)]}{(x-3)^4}$$

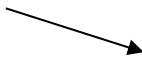
$$= \frac{2(x-4)}{(x-3)^3}$$

For critical numbers,

$$0 = \frac{2(x-4)}{(x-3)^3}$$

$$x = 4$$

Also, $f'(x)$ is undefined for $x = 3$.

	$x < 3$	$3 < x < 4$	$x > 4$
Sign of $f'(x)$	+	-	+
Increase/Decrease for $f(x)$			

Maximum and Minimum Points

Local minimum at $(4, 0)$.

Concavity

$$f'(x) = \frac{2(x-4)}{(x-3)^3}$$

$$\begin{aligned} f''(x) &= \frac{2(x-3)^3 - 3(x-3)^2(2)(x-4)}{(x-3)^6} \\ &= \frac{2(x-3)^2[x-3-3(x-4)]}{(x-3)^6} \\ &= \frac{2(-2x+9)}{(x-3)^4} \\ &= \frac{-2(2x-9)}{(x-3)^4} \end{aligned}$$

For possible inflection points,

$$0 = \frac{-2(2x-9)}{(x-3)^4}$$

$$x = 4.5$$

Also, $f''(x)$ is undefined for $x = 3$.

	$x < 3$	$3 < x < 4.5$	$x > 4.5$
Sign of $f''(x)$	+	+	-
Concavity of $f(x)$	Up	Up	Down

Inflection Points

Inflection point at (4.5, 0.1)

