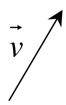
MULTIPLYING A VECTOR BY A SCALAR

Multiplying a Vector by a Scalar

For the following vector $\vec{v}$, find the sum $\vec{v} + \vec{v} + \vec{v}$.



As you might expect, $\vec{v} + \vec{v} + \vec{v} = 3\vec{v}$, where $3\vec{v}$ is a vector in the same direction as $\vec{v}$ with 3 times the magnitude.

A vector $-2\vec{v}$ would be in the opposite direction to $\vec{v}$ with twice the magnitude.

The operation of multiplying a vector by a scalar is called **scalar multiplication**.

Scalar Multiplication

Let $\vec{v}$ be a vector and let k be a scalar.

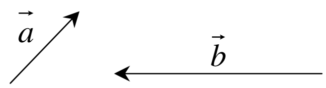
Magnitude
- $k\vec{v}$ is a vector that is $|k|$ times as long as $\vec{v}$.

Direction
- If $k > 0$, $k\vec{v}$ has the ____________ direction as $\vec{v}$.
- If $k < 0$, $k\vec{v}$ has the ____________ direction of $\vec{v}$.
- If $k = 0$, $k\vec{v}$ is the ___________________ .

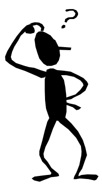
Note the similarities between multiplication in arithmetic and scalar multiplication of vectors.

Example

For the given vectors $\vec{a}$ and $\vec{b}$, sketch $2\vec{a}$, $-3\vec{b}$ and $\frac{1}{2}\vec{b}$.



Two vectors are said to be **collinear** if they lie on a straight line when arranged tail-to-tail. Notice that if one vector is a scalar multiple of another vector, the two vectors will be collinear.



What about the zero vector?

Linear Combinations of Vectors

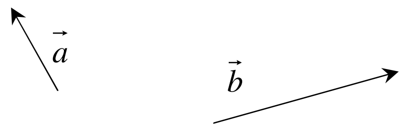
Scalar multiplication of vectors is often combined with vector addition and subtraction to give **linear combinations**.

Linear Combinations of Vectors

A linear combination of the vectors $\vec{u}$ and $\vec{v}$ has the form $a\vec{u} + b\vec{v}$, where a and b are scalars.

Example

For the given vectors $\vec{a}$ and $\vec{b}$, sketch $-3\vec{a} + \frac{1}{2}\vec{b}$.

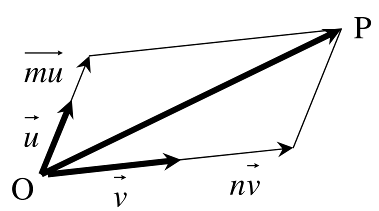


Example

ABCD is a parallelogram with *P* and *Q* the midpoints of *AB* and *DA* respectively. If $\vec{u} = \overrightarrow{BP}$ and $\vec{v} = \overrightarrow{AQ}$, express the following vectors in terms of $\vec{u}$ and $\vec{v}$.

a) $\overrightarrow{CD}$ b) $\overrightarrow{BD}$ c) $\overrightarrow{PD}$ d) $\overrightarrow{AC}$

If $\vec{u}$ and $\vec{v}$ are non-zero, non-collinear vectors, then any vector $\overrightarrow{OP}$ in the plane containing $\vec{u}$ and $\vec{v}$ can be expressed as a unique linear combination of $\vec{u}$ and $\vec{v}$.



$$\overrightarrow{OP} = m\vec{u} + n\vec{v}$$