Polynomial Functions in Factored Form

For each of the following, state the zeros and the end behaviour.

a) $f(x) = (x+7)(x+4)(x-5)$

- Zeros are -7, -4 and 5
- Function is cubic (degree 3)
- Leading coefficient is 1
- End behaviour:
  As $x \to \infty$, $f(x) \to \infty$
  As $x \to -\infty$, $f(x) \to -\infty$

b) $f(x) = -2(x+3)(x-2)(2x-5)^2$
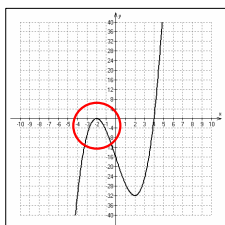
- Zeros are -3, 2 and $\frac{5}{2}$
- Function is quartic (degree 4)
- Leading coefficient is -8
- End behaviour:
  As $x \to \infty$, $f(x) \to -\infty$
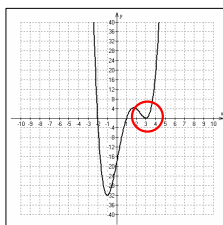 As $x \to -\infty$, $f(x) \to -\infty$

Inspection

Inspect the graphs of the following functions. What do you notice about the zeros corresponding to squared factors?
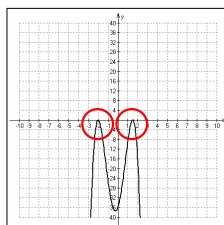
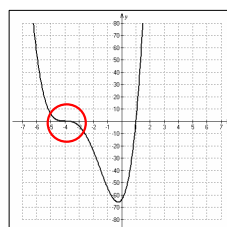$f(x) = (x+2)^2(x-4)$ $f(x) = (x+2)(x-1)(x-3)^2$ $f(x) = -(x+2)^2(2x-3)^2$

The zeros corresponding to squared factors are also turning points. The graph has a parabolic shape near these points. The x-axis is tangent to the curve at these points.
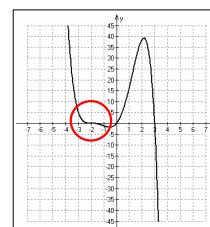
Inspection

Inspect the graphs of the following functions. What do you notice about the zeros corresponding to cubed factors?
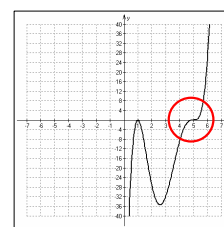
$f(x) = (x-1)(x+4)^3$ $f(x) = -0.3x(x-3)(x+2)^3$ $f(x) = (x-1)^2(x-5)^3$

The graph resembles the graph of $y = x^3$ at the zeros corresponding to cubed factors.

Some examples…
The following are to be completed on a separate page.

1) Sketch the graph of the following functions, considering intercepts, end behaviour and any necessary additional points.

   a) $f(x) = -0.5(x+1)(x+3)(x-2)^2$ b) $f(x) = x^4 + 2x^3$

2) Write the equation of a cubic function that has zeros at -2, 3, and $\frac{2}{5}$ and that has a y-intercept of 6.

3) For the given graph, write the equation of the function and state its domain and range.
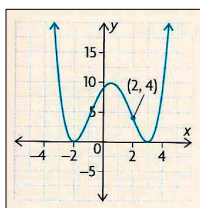
A little bit of terminology…

ORDER:
The exponent to which a factor in an algebraic expression is raised is called its *order*. For example, in $f(x) = (x-3)^2(x-1)$, the order of $(x-3)$ is 2 and the order of $(x-1)$ is 1.

FAMILY OF POLYNOMIAL FUNCTIONS:

A *family of polynomial functions* is a set of polynomial functions whose equations have the same degree and whose graphs have common characteristics. For example, all functions of the form $y = a(x-3)(x+7)(x+9)$, where $a \neq 0$.